<u>Ex 1 :</u> $\int x^n = \frac{1}{n+1} x^{n+1}$

car $\left(\frac{1}{n+1} x^{n+1}\right)' = \frac{1}{\cancel{n+1}} \times \cancel{(n+1)} x^n = x^n$.

$$\int_0^1 x^n \, dx = \left[\frac{1}{n+1} x^{n+1}\right]_0^1 = \frac{1}{n+1}\left[x^{n+1}\right]_0^1$$

$$= \frac{1}{n+1}\left(1^{n+1} - 0^{n+1}\right)$$

$$= \frac{1}{n+1}$$

$$= \begin{cases} 1 & \text{si} \quad n=0 \\ 1/2 & \text{si} \quad n=1 \\ 1/3 & \text{si} \quad n=2 \end{cases}.$$

<u>Ex 2 :</u> $a, b, c \in \mathbb{R}$,

$$F(t) = \int_0^t ax^2 + bx + c \, dx$$

$$F(r) = a\int_0^r x^2\,dx + b\int_0^r x\,dx + c\int_0^1 1\,dx$$

$$= a\left[\frac{x^3}{3}\right]_0^r + b\left[\frac{x^2}{2}\right]_0^r + c\left[x\right]_0^r$$

$$= \frac{a t^3}{3} + \frac{b t^2}{2} + ct$$

$$F'(r) = a t^2 + bt + c$$

Ex 3

$$I = \int_{x=0}^{1}\int_{y=0}^{2} 2x+y \; dy\,dx$$

$$= \int_{x=0}^{1} \left[2xy + \frac{y^2}{2}\right]_{y=0}^{2} dx$$

$$= \int_{x=0}^{1} 4x+2 \; dx = \left[2x^2+2x\right]_0^1 = 4$$

$$J = \int_{x=0}^{1} \int_{y=0}^{1} x\, y \, dy \, dx$$

$$= \int_{x=0}^{1} x \left(\int_{y=0}^{1} y \, dy \right) dx$$

$$= \int_{x=0}^{1} x \left[\frac{y^2}{2} \right]_0^1 dx$$

$$= \int_{x=0}^{1} x \times \frac{1}{2} \, dx$$

$$= \frac{1}{2} \left[\frac{x^2}{2} \right]_0^1 = \frac{1}{4}$$

Ex 4 : (On prend $0 \leq s \leq t$)

$$I = \int_s^t f(y) \, dy \qquad y = \sqrt{x}$$

$y = t \iff x = t^2$

$y = s \iff x = s^2$

$$dy = \frac{dx}{2\sqrt{x}}$$

$$I = \int_{s^2}^{t^2} f(\sqrt{x}) \times \frac{dx}{2\sqrt{x}}$$

$$= \frac{1}{2} \int_{s^2}^{r^2} \frac{f(\sqrt{x})}{\sqrt{x}}\, dx$$

<u>Eg</u> : $I = \int_0^2 y e^{-y^2}\, dy$

$$= \int_0^2 f(y)\, dy \quad \left(\text{où } f(y) = y e^{-y^2} \right)$$

$$= \frac{1}{2} \int_0^4 \frac{f(\sqrt{x})}{\sqrt{x}}\, dx$$

$$= \frac{1}{2} \int_0^4 \frac{\cancel{\sqrt{x}}\, e^{-\sqrt{x}^2}}{\cancel{\sqrt{x}}}\, dx$$

$\left(\underline{\text{Note}} : \sqrt{x}^2 = |x| \right.$
$\left. = x \text{ si } x \geq 0 \ldots \right)$

$$= \frac{1}{2} \int_0^4 e^{-x}\, dx = \frac{1}{2} \left[-e^{-x} \right]_0^4 = \frac{1}{2} \left[e^{-x} \right]_4^0 = \frac{1}{2} \left(1 - e^{-4} \right)$$

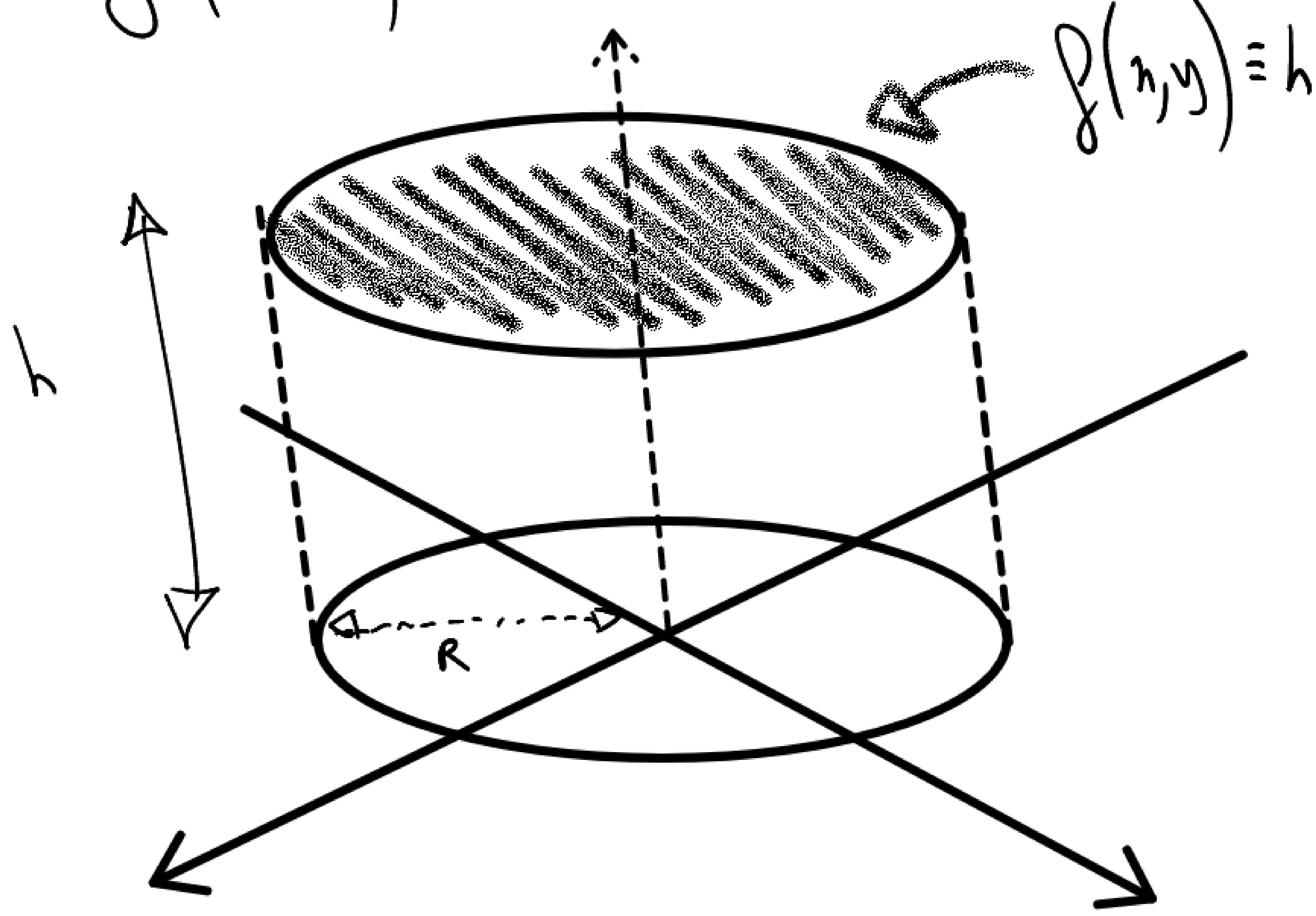
Note : Exemple artificiel car on peut directement voir :

$$\int y e^{-y^2} dy = -\frac{1}{2} e^{-y^2} \dots$$

Ex 5 :

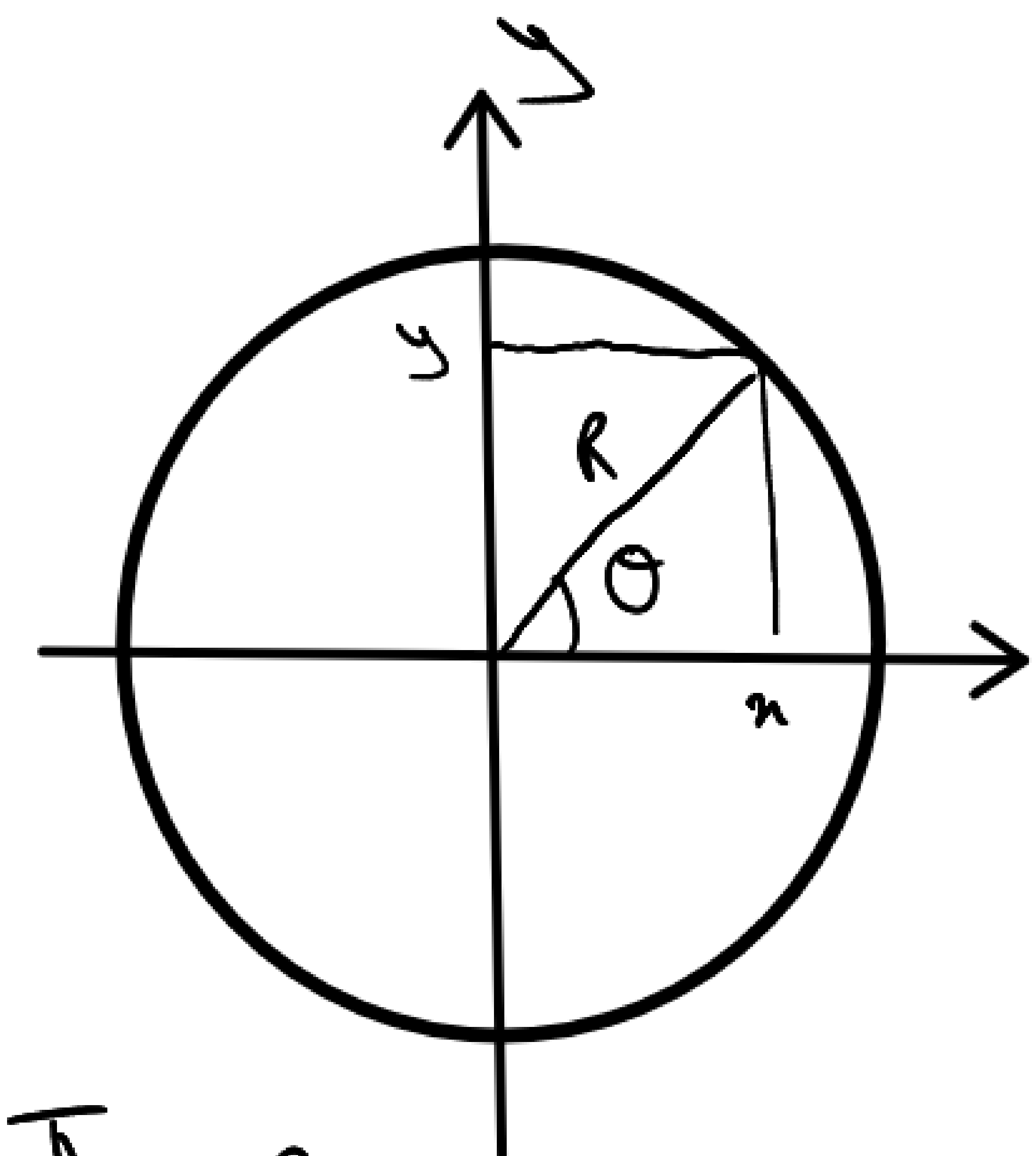
a) $\mathcal{D} = \{ (x,y) \in \mathbb{R}^2 \, / \, \|(x,y)\| < R \}$ $\quad (R > 0)$

$f(x,y) = h$ $\quad (h > 0)$



$$I = \iint_{\mathcal{D}} f(x,y)\, dy\, dx = \underbrace{\iint_{\mathcal{D}} h\, dy\, dx}_{\text{Volume cylindre}} = \underbrace{h}_{\text{Hauteur}} \times \underbrace{\iint_{\mathcal{D}} dy\, dx}_{\text{Aire du disque}}$$

$$I = h \times \iint_{D} 1 \, dy \, dx$$



$$-\pi \leq \theta \leq \pi$$
$$0 \leq r \leq R$$

$$= h \int_{-\pi}^{\pi} \int_{0}^{R} r \, dr \, d\theta$$

$$= h \times \int_{-\pi}^{\pi} \left[\frac{r^2}{2} \right]_{0}^{R} d\theta$$

$$= h \times \int_{-\pi}^{\pi} \frac{R^2}{2} \, d\theta$$

$$= h \times \frac{R^2}{2} \times \int_{-\pi}^{\pi} 1 \, d\theta$$

$$= h \times \frac{R^2}{2} \left[\theta \right]_{-\pi}^{\pi}$$

$$I = h \times \frac{R^2}{2}\left(\pi - (-\pi)\right)$$

$$= h \times \frac{R^2}{\cancel{2}} \times \cancel{2}\pi$$

$$= h \times \underbrace{\pi R^2}_{\text{Aire du disque!}}$$

$$\left(\mathcal{A}_{cylindre} = \text{Hauteur} \times \mathcal{A}_{base} \right)$$

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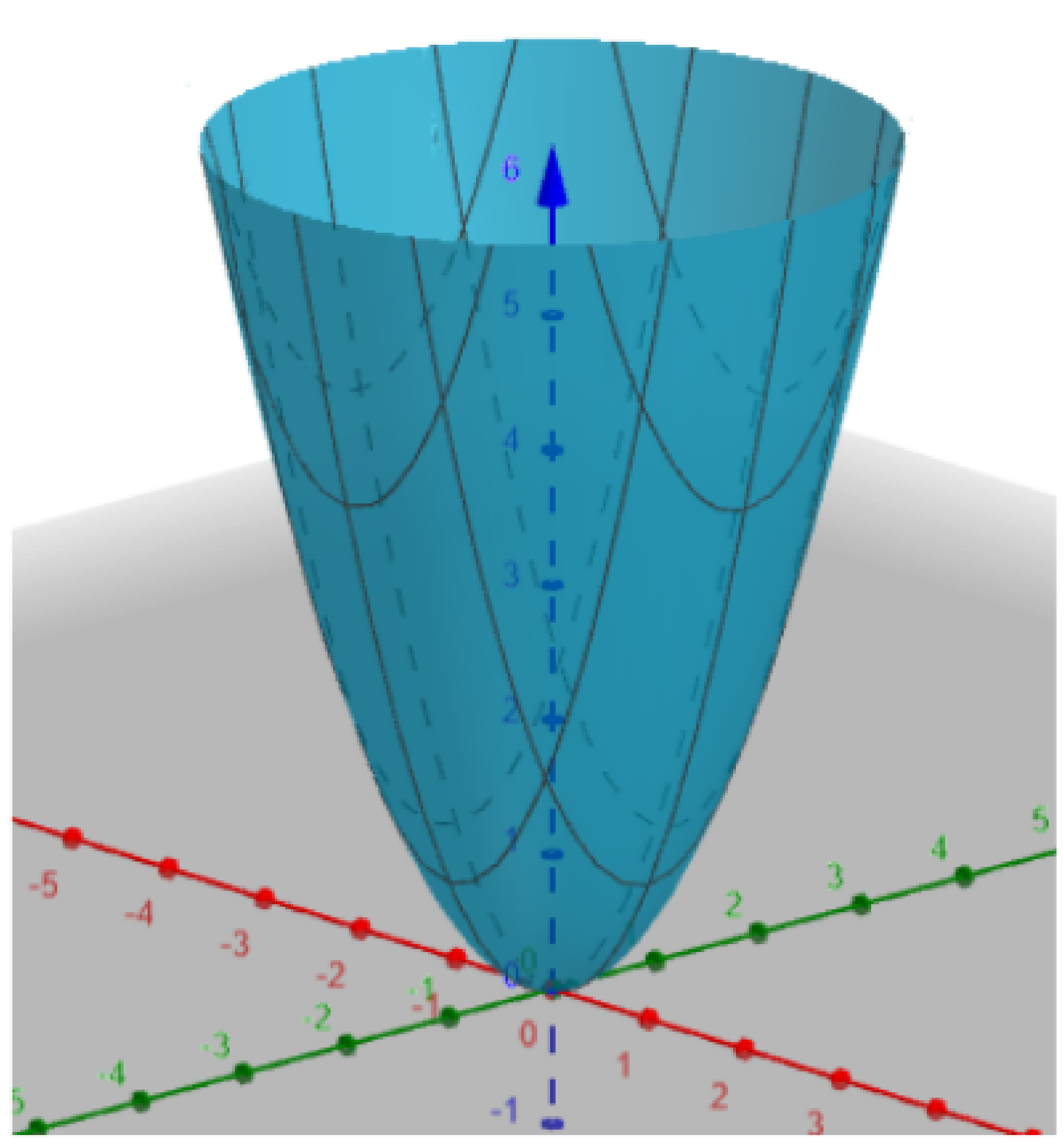
b) $f(x,y) = x^2 + y^2$



$$J = \iint_D x^2 + y^2$$

$$\left| \begin{array}{l} x = r\cos(\theta) \\ y = r\sin\theta \end{array} \right.$$

$$x^2 = r^2 \cos^2(\theta)$$

$$+ \left(y^2 = r^2 \sin(\theta) \right)$$

$$x^2 + y^2 = r^2 \underbrace{\left(\cos^2(\theta) + \sin^2(\theta) \right)}_{=1 \text{ (Pythagore)}}$$

$$= \int_{-\pi}^{\pi} \int_0^R r^2 \times r\,dr\,d\theta$$

$$= \int_{-\pi}^{\pi} \int_0^R r^3\,dr\,d\theta = \int_{-\pi}^{\pi} \left[\frac{r^3}{3} \right]_0^R d\theta$$

$$= \int_{-\pi}^{\pi} \frac{R^3}{3}\,d\theta = \frac{R^3}{3} \int_{-\pi}^{\pi} d\theta$$

$$= \frac{R^3}{3} \times \left[\theta \right]_{-\pi}^{\pi} = \frac{R^3}{3} \times 2\pi$$

$$= \frac{2R^3\pi}{3}$$